VBTL

Complexe getallen

[uitgebreid]

Philip Bogaert
Filip Geeurickx
Roger Van Nieuwenhuyze
Erik Willockx

Voor het GO! onderwijs van de
Vlaamse Gemeenschap bewerkt door:
Paul Boogmans
Hugo Dirks
Paul Verbelen

cartoons:
Dave Vanroye

5 • 6

die Keure

INHOUD

Complexe getallen

1

1.4 Vierkantsvergelijkingen in ℂ

Oef 1 blz 28

Bereken in $\mathbb{C}$.

a $(2-i)+(3+4i)$

$= 2 - i + 3 + 4i$

$= 5 + 3i$

b $(4i-2)-(2+5i)$

$= 4i - 2 - 2 - 5i$

$= -4 - i$

c $(7+6i)+2(1+i)$

$= 7 + 6i + 2 + 2i$

$= 9 + 8i$

d $3(4-i)-4(i+3)$

$= 12 - 3i - 4i - 12$

$= -7i$

e $(2-3i)(1-i)$

$= 2 - 2i - 3i + 3i^2$

$= 2 - 2i - 3i - 3$

$= -1 - 5i$

g $(2+i)(-2+i)$

$= i^2 - 4$

$= -1 - 4$

$= -5$

h $(4-i)(3-2i)-(2+3i)(4i-1)$

$= 12 - 8i - 3i + 2i^2 - \left(8i - 2 + 12i^2 - 3i\right)$

$= 122 - 11i - 2 - 5i + 2 + 12$

$= 24 - 16i$

i $(2+i)(2-i)-2(1+i)(3-i)$

$$= 4-i^2-2\left(3-i+3i-i^2\right)$$

$$= 4+1-6-4i-2$$

$$= -3-4i$$

j $(3-i)(1+3i)(2-2i)$

$$= \left(3+9i-i-3i^2\right)(2-2i)$$

$$= (8+8i)(2-2i)$$

$$= 12-12i+16i-16i^2$$

$$= 28+4i$$

k $(1+2i)(1-3i)(1+4i)$

$$= \left(1-3i+2i-6i^2\right)(1+4i)$$

$$= (7-i)(1+4i)$$

$$= 7+28i-i-4i^2$$

$$= 11+27i$$

l $(1-i)^2$

$$= 1-2i+i^2$$

$$= 1-2i-1$$

$$= -2i$$

m $(2+3i)^2$

$$= 4+12i+9i^2$$

$$= 4+12i-9$$

$$= -5+12i$$

n $(5i)^3$

$$= -125i$$

o $(-3i)^4$

$$= 81i^4$$

$$= 81$$

p $(-2i)^5$

$$= -32i^5$$

$$= -32i$$

q $(2+i)^3$

$= 8+3\cdot 4i+3\cdot 2i^2+i^3$

$= 8+12i-6-i$

$= 2+11i$

r $(6-i)(2+3i)-(4+2i)^2$

$= 12+18i-2i-3i^2-\left(16+16i+4i^2\right)$

$= 12+16i+3-12-16i$

$= 3$

s $2i(1+i)^2-i(2-i)^2$

$= 2i\left(1+2i+i^2\right)-i\left(4-4i+i^2\right)$

$= 2i\cdot 2i-i(3-4i)$

$= 4i^2-3i+4i^2$

$= -8-3i$

t $2(1-7i)^2-2(2-i)(3+i)(4-i)$

$= 2\left(1-14i+49i^2\right)+\left(6+2i-3i-i^2\right)(4-i)$

$= 2(-48-14i)-(7-i)(4-i)$

$= -96-28i-28+7i+4i-i^2$

$= -123-17i$

u $(1-i)^4$

$= \left(1-2i+i^2\right)^2$

$= (-2i)^2$

$= 4i^2$

$= -4$

v $i^7(3-4i)+i^5(1+2i)^2-i^3+i$

$= -i(3-4i)+i\left(1+4i+4i^2\right)+i+i$

$= -3i+4i^2-3i+4i^2+2i$

$= -8-4i$

Oef 2 blz 28

Bereken in $\mathbb{C}$.

a $\overline{2-3i}$

$= 2 + 3i$

b $\overline{-7i}$

$= 7i$

c $\overline{3-i}+\overline{4+2i}$

$= 3 + i + 4 - 2i$

$= 7 - i$

d $\overline{(1+3i)(4+5i)}$

$=(1-3i)(4-5i)$

$=4-5i-12i+15i^2$

$=-11-17i$

e $\overline{2i(5-i)}$

$=-2i(5+i)$

$=-10i-2i^2$

$=2-10i$

f $(2+3i)\overline{(3-2i)}$

$=(2+3i)(3+2i)$

$=6+4i+9i+6i^2$

$=13i$

g $\overline{(4-i)\overline{(1+2i)}}$

$=\overline{(4-i)(1-2i)}$

$=\overline{4-8i-i+2i^2}$

$=\overline{2-9i}$

$=2+9i$

h $\overline{i\cdot\overline{(1+i)^2}}$

$= \overline{i\cdot(1-i)^2}$

$= \overline{i(1-2i+i^2)}$

$= \overline{i\cdot(-2i)}$

$= \overline{-2i^2}$

$= \overline{2}$

$= 2$

Oef 3 blz 28

Gegeven: $z_1 = 3-2i$

$z_2 = -1-i$

$z_3 = 4+5i$

Bereken in $\mathbb{C}$:

a $z_1+z_2+z_3$

$= 3-2i+(-1-i)-(4+5i)$

$= 3-2i-1-i-4-5i$

$= -2-8i$

b $z_1\cdot z_2$

$= (3-2i)\cdot(-1-i)$

$= -3-3i+2i+2i^2$

$= -5-i$

c $\overline{z_2}$

$= \overline{-1-i}$

$= -1+i$

d z_3^2

$= (4+5i)^2$

$= 16+40i+25i^2$

$= -9+40i$

e $z_1^2-2\cdot z_2\cdot z_3+i\left(z_1+3z_2\right)$

$= (3-2i)^2-2(-1-i)(4+5i)+i(3-2i-3-3i)$

$= 9-12i+4i^2-2\left(-4-5i-4i-5i^2\right)+i(-5i)$

$= 9-12i-4-2+18i+5$

$= 8+6i$

f $\quad \left(z_1-\overline{z_2}\right)\left(\overline{z_3}-i\cdot z_2\right)$

$$=\left(3-2i-\overline{(-1-i)}\right)\cdot\left(\overline{4+5i}-i(3-2i)\right)$$

$$=\left(3-2i-(-1+i)\right)(4-5i-3i-2)$$

$$=(4-3i)(2-8i)$$

$$=8-32i-6i+24i^2$$

$$=-16-38i$$

Oef 4 blz 29

Gegeven : $\quad z_1=5+i$

$z_2=1-6i$

$z_3=3i+4$

Bereken in $\mathbb{C}$:

a $\quad z_1-\overline{z_2}+2z_3$

$$=5+i-\overline{(1-6i)}+2(3i+4)$$

$$=5+i-(1+6i)+6i+8$$

$$=5+i-1-6i+6i+8$$

$$=12+i$$

b $\quad \overline{z_1\cdot z_2}$

$$=\overline{(5+i)(1-6i)}$$

$$=(5-i)(1+6i)$$

$$=5+30i-i-6i^2$$

$$=11+29i$$

c $\quad \overline{z_3-\overline{z_1}}$

$$=\overline{3i+4-\overline{(5+i)}}$$

$$=\overline{3i+4-(5-i)}$$

$$=\overline{3i+4-5+i}$$

$$=\overline{-1+4i}$$

$$=-1-4i$$

d $\left(\overline{z_1}+2z_2\right)^2-z_3\cdot\overline{z_3}$

$$=\left(\overline{(5+i)}+2(1-6i)\right)^2-(3i+4)\cdot\overline{(3i+4)}$$

$$=(5-i+2-12i)^2-(3i+4)(-3i+4)$$

$$=(7-13i)^2-(16-9i^2)$$

$$=49-182i+169i^2-25$$

$$=-145-182i$$

e $\mathrm{Re}\left(z_1-\overline{2z_2}\right)$

$$z_1-\overline{2z_2}=5+i-\overline{2(1-6i)}$$

$$=5+i-2(1+6i)$$

$$=5+i–2-12i$$

$$=3-11i$$

$$\mathrm{Re}\left(z_1-\overline{2z_2}\right)=3$$

f $\mathrm{Im}\left(\overline{\overline{z_3}}+z_1\right)$

$$\overline{\overline{z_3}}+\overline{z_1}=z_3+\overline{z_1}$$

$$=3i+4+\overline{5+i}$$

$$=3i+4+5-i$$

$$=9+2i$$

$$\mathrm{Im}\left(\overline{\overline{z_3}}+\overline{z_1}\right)=2$$

Oef 5 blz 29

Bereken in $\mathbb{C}$.

a $\dfrac{2}{3+2i}$

$$=\frac{2(3-2i)}{(3+2i)(3-2i)}$$

$$=\frac{6-4i}{9-4i^2}$$

$$=\frac{6-4i}{13}$$

b $\dfrac{-1}{5+2i}$

$$=\frac{-1(5-2i)}{(5+2i)(5-2i)}$$

$$=\frac{-5+2i}{25-4i^2}$$

$$=\frac{-5+2i}{29}$$

c $\frac{2-4i}{1+i}$

$$=\frac{(2-4i)(1-i)}{(1+i)(1-i)}$$

$$=\frac{2-2i-4i+4i^2}{1-i^2}$$

$$=\frac{-2-6i}{2}$$

$$=-1-3i$$

d $\frac{7+4i}{i}$

$$=\frac{(7+4i)(-i)}{i(-i)}$$

$$=\frac{-7i-4i^2}{-i^2}$$

$$=4-7i$$

e $\frac{3-2i}{3+2i}$

$$=\frac{(3-2i)(3-2i)}{(3+2i)(3-2i)}$$

$$=\frac{9-12i+4i^2}{9-4i^2}$$

$$=\frac{9-12i-4}{9+4}$$

$$=\frac{5-12i}{13}$$

f $\frac{3i-1}{2-i}$

$$=\frac{(3i-1)(2+i)}{(2-i)(2+i)}$$

$$=\frac{6i+3i^2-2-i}{4-i^2}$$

$$=\frac{-5+5i}{5}$$

$$=-1+i$$

g $\frac{1}{i}+i$

$$=\frac{1+i^2}{i}$$

$$=0$$

h $\quad \frac{1-3i}{2+3i}+\frac{2-3i}{1+3i}$

$$=\frac{(1-3i)(2-3i)}{(2+3i)(2-3i)}+\frac{(2-3i)(1-3i)}{(1+3i)(1-3i)}$$

$$=\frac{2-9i+9i^2}{4-9i^2}+\frac{2-9i+9i^2}{1-9i^2}$$

$$=\frac{-7-9i}{13}+\frac{-7-9i}{10}$$

$$=\frac{-161}{130}-\frac{207i}{130}$$

i $\quad \frac{1+i}{1-i}-\frac{1-i}{1+i}$

$$=\frac{(1+i)^2}{(1-i)(1+i)}-\frac{(1-i)^2}{(1+i)(1-i)}$$

$$=\frac{1+2i-1-(1-2i-1)}{1+1}$$

$$=\frac{4i}{2}$$

$$=2i$$

j $\quad \frac{2i+5}{3-i}+\frac{4+i}{1+3i}$

$$=\frac{(2i+5)(3+i)}{(3-i)(3+i)}+\frac{(4+i)(1-3i)}{(1+3i)(1-3i)}$$

$$=\frac{6i-2+15+5i}{9+1}+\frac{4-12i+i+3}{1+9}$$

$$=\frac{11i+13}{10}+\frac{7-11i}{10}$$

$$=\frac{20}{10}$$

$$=2$$

k $\quad \frac{-i}{1-2i}+\frac{\overline{7-2i}}{i}$

$$=\frac{-i(1+2i)}{(1-2i)(1+2i)}+\frac{7+2i}{i}$$

$$=\frac{-i+2}{1+4}-7i+2$$

$$=\frac{-i+2}{5}+\frac{-35i+10}{5}$$

$$=\frac{12-36i}{5}$$

l $\dfrac{(2-i)^2}{(1+2i)^2}$

$$= \frac{4-4i+i^2}{1+4i+4i^2}$$

$$= \frac{3-4i}{-3+4i}$$

$$= -1$$

m $\dfrac{(2-3i)(4+5i)}{(2-5i)(3+4i)}$

$$= \frac{8+10i-12i+15}{6+8i-15i+20}$$

$$= \frac{(23-2i)(26+7i)}{(26-7i)(26+7i)}$$

$$= \frac{598+161i-52i+14}{676+49}$$

$$= \frac{612+109i}{725}$$

n $\left(\dfrac{1+i\sqrt{2}}{1-i\sqrt{2}}\right)^2 + \left(\dfrac{1-i\sqrt{2}}{1+i\sqrt{2}}\right)^2$

$$= \frac{1+2i\sqrt{2}-2}{1-2i\sqrt{2}-2} + \frac{1-2i\sqrt{2}-2}{1+2i\sqrt{2}-2}$$

$$= \frac{2i\sqrt{2}-1}{-2i\sqrt{2}-1} + \frac{-2i\sqrt{2}-1}{2i\sqrt{2}-1}$$

$$= \frac{\left(2i\sqrt{2}-1\right)^2 + \left(-2i\sqrt{2}-1\right)^2}{\left(-2i\sqrt{2}-1\right)\left(2i\sqrt{2}-1\right)}$$

$$= \frac{-8-4i\sqrt{2}+1-8+4i\sqrt{2}+1}{8+1}$$

$$= -\frac{14}{9}$$

Oef 6 blz 29

Gegeven : $z_1 = 2 - i$

$z_2 = 1 + 2i$

$z_3 = -3 + i$

Bereken in $\mathbb{C}$:

a $\quad \dfrac{z_1}{z_2}$

$$= \frac{2-i}{1+2i}$$

$$= \frac{(2-i)(1-2i)}{(1+2i)(1-2i)}$$

$$= \frac{2-4i-i-2}{1+4}$$

$$= -\frac{5i}{5}$$

$$= -i$$

b $\quad \dfrac{1}{z_3} - \dfrac{1}{z_2} + \dfrac{1}{z_1}$

$$= \frac{1}{-3+i} - \frac{1}{1+2i} + \frac{1}{2-i}$$

$$= \frac{-3-i}{9+1} - \frac{1-2i}{1+4} + \frac{2+i}{4+1}$$

$$= \frac{-3-i-2+4i+4+2i}{10}$$

$$= \frac{-1+5i}{10}$$

c $\quad \dfrac{z_3}{z_1} - \dfrac{z_2}{\overline{z_1}}$

$$= \frac{-3+i}{2-i} - \frac{1+2i}{2+i}$$

$$= \frac{(-3+i)(2+i)}{4+1} - \frac{(1+2i)(2-i)}{4+1}$$

$$= \frac{-6-3i+2i-1-(2-i+4i+2)}{5}$$

$$= \frac{-11-4i}{5}$$

d $\frac{\overline{z_3}}{z_3}+\frac{z_3}{\overline{z_3}}$

$$=\frac{-3-i}{-3+i}+\frac{-3+i}{-3-i}$$

$$=\frac{(-3-i)^2}{9+1}+\frac{(-3+i)^2}{9+1}$$

$$=\frac{9+6i-1+9-6i-1}{10}$$

$$=\frac{16}{10}$$

$$=\frac{8}{5}$$

Oef 7 blz 29

Los op in $\mathbb{C}$.

a $(2-i)z+3-4i=0$

$$\Leftrightarrow (2-i)z=-3+4i$$

$$\Leftrightarrow z=\frac{-3+4i}{2-i}$$

$$\Leftrightarrow z=-2+i$$

$$V=\{-2+i\}$$

b $(1+i)(z-2)=(3+4i)(1-iz)$

$$\Leftrightarrow (1+i)z-2(1+i)=3+4i-i(3+4i)z$$

$$\Leftrightarrow \big((1+i)+i(3+4i)\big)z=3+4i+2(1+i)$$

$$\Leftrightarrow (1+i+3i-4)z=5+6i$$

$$\Leftrightarrow (-3+4i)z=5+6i$$

$$\Leftrightarrow z=\frac{5+6i}{-3+4i}$$

$$\Leftrightarrow z=\frac{9-38i}{25}$$

$$V=\left\{\frac{9-38i}{25}\right\}$$

c $(2+i)(z-i)-(3-i)((z+i)+(4+i)(iz)=0$

$\Leftrightarrow (2+i)z-2i+1-(3-i)z-3i-1+(4i-1)z=0$

$\Leftrightarrow (2+i-3+i+4i-1)z-5i=0$

$\Leftrightarrow (-2+6i)z=5i$

$\Leftrightarrow z=\dfrac{5i}{-2+6i}$

$\Leftrightarrow z=\dfrac{3-i}{4}$

$V=\left\{\dfrac{3-i}{4}\right\}$

d $8+(1+i)^2 z=(2-3i)(3+2i)+iz$

$\Leftrightarrow 2iz-iz=12-5i-8$

$\Leftrightarrow iz=4-5i$

$\Leftrightarrow z=-5-4i$

$V=\{-5-4i\}$

e $(5-2i)(z+2(6+i)\bar{z}=1+28i$

stel $z=x+yi \quad (x,y\in \mathrm{IR})$

$\Leftrightarrow (5-2i)(x+yi)+(12+2i)(x-yi)=1+28i$

$\Leftrightarrow 5x+5yi-2xi+2y+12x-12yi+2xi+2y=1+28i$

$\Leftrightarrow (17x+4y)+(-7y)i=1+28i$

$\Leftrightarrow \begin{cases}17x+4y=1\\ -7y=28\end{cases}$

$\Leftrightarrow \begin{cases}x=1\\ y=-4\end{cases}$

Hieruit volgt: $z=1-4i$

$V=\{1-4i\}$

f $(1-8i)z+22i=72-(3+4i)\overline{z}$

stel $z=x+yi \qquad (x,y\in \mathrm{IR})$

$\Leftrightarrow (1-8i)(x+yi)+22i=72-(3+4i)(x-yi)$

$\Leftrightarrow x+yi-8xi+8y+3x-3yi+4xi+4y=72-22i$

$\Leftrightarrow (4x+12y)+(-4x-2y)i=72-22i$

$$\Leftrightarrow \begin{cases} 4x+12y=72 \\ -4x-2y=-22 \end{cases}$$

$$\Leftrightarrow \begin{cases} x=3 \\ y=5 \end{cases}$$

Hieruit volgt: $z=3+5i$

$V=\{3+5i\}$

Oef 8 blz 30

a Bereken $i^3, i^4, i^5, i^6, i^7, i^8, i^9, i^{10}$

$i^3=i^2\cdot i=-1\cdot i=-i$

$i^4=i^3\cdot i=-i\cdot i=-i^2=1$

$i^5=i^4\cdot i=1\cdot i=i$

$i^6=i^5\cdot i=i\cdot i=i^2=-1$

$i^7=i^6\cdot i=-1\cdot i=-i$

$i^8=i^7\cdot i=-i\cdot i=-i^2=1$

$i^9=i^8\cdot i=1\cdot i=i$

$i^{10}=i^9\cdot i=i\cdot i=i^2=-1$

b Bereken $i^{4k}, i^{4k+1}, i^{4k+2}, i^{4k+3}$

$i^{4k}=\left(i^4\right)^k=1^k=1$

$i^{4k+1}=\left(i^{4k}\right)\cdot i=1\cdot i=i$

$i^{4k+2}=\left(i^{4k}\right)\cdot i^2=1\cdot(-1)=-1$

$i^{4k+3}=\left(i^{4k}\right)\cdot i^3=1\cdot(-i)=-i$

c Bereken i^{-1}, i^{-2}, i^{-3}, i^{-4}, i^{-5}, i^{-6}

$$i^{-1} = \frac{1}{i} = \frac{i^4}{i} = i^3 = -i$$

$$i^{-2} = \frac{1}{i^2} = \frac{1}{-1} = -1$$

$$i^{-3} = \frac{1}{i^3} = \frac{i^4}{i^3} = i$$

$$i^{-4} = \frac{1}{i^4} = \frac{1}{(-1)^2} = 1$$

$$i^{-5} = \frac{1}{i^5} = \frac{1}{i^4 \cdot i} = \frac{1}{i} = -i$$

$$i^{-6} = \frac{1}{i^6} = \frac{1}{-1} = -1$$

d Bereken i^n waarin n het huidige jaartal voorstelt.

$$i^{2018} = \left(i^2\right)^{1009} = (-1)^{1009} = -1$$

Oef 9 blz 30

Bepaal de vierkantswortel uit:

a -9

$x + yi$ is een vierkantswortel van -9

$$\Leftrightarrow (x+yi)^2 = -9$$

$$\Leftrightarrow (x+yi)^2 = 9i^2$$

$$\Leftrightarrow (x+yi)^2 = (3i)^2$$

$$\Leftrightarrow x+yi = 3i \quad \text{of} \quad x+yi = -3i$$

Antwoord : in $\mathbb{C}$ heeft -9 twee tegengestelde vierkantswortels: $3i$ en $-3i$

b $2i$

$x + yi$ is een vierkantswortel van $2i$

$$\Leftrightarrow (x+yi)^2 = 2i$$

$$\Leftrightarrow x^2 + 2xyi + y^2 i^2 = 2i$$

$$\Leftrightarrow \begin{cases} x^2 - y^2 = 0 \\ 2xy = 2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2 - y^2 = 0 \\ xy = 1 \end{cases}$$

$$\underset{x\neq 0}{\Leftrightarrow} \begin{cases} y = \dfrac{1}{x} \\ x^2 - \left(\dfrac{1}{x}\right)^2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = \dfrac{1}{x} \\ x^4 - 1 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = \dfrac{1}{x} \\ x^2 - 1 = 0 \end{cases} \quad \text{of} \quad \begin{cases} y = \dfrac{1}{x} \\ \cancel{x^2+1} = 0 \quad (x^2 \in \mathbb{R} \Rightarrow x^2 > 0) \end{cases}$$

$$\Leftrightarrow \begin{cases} y = \dfrac{1}{x} \\ x = \pm 1 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = 1 \\ x = 1 \end{cases} \quad \text{of} \quad \begin{cases} y = -1 \\ x = -1 \end{cases}$$

$$\Leftrightarrow x+yi = 1+i \quad \text{of} \quad x+yi = -1-i$$

Antwoord : in $\mathbb{C}$ heeft $2i$ twee tegengestelde vierkantswortels: $1+i$ en $-1-i$

c $\quad -3-4i$

$x + yi$ is een vierkantswortel van $-3-4i$

$\Leftrightarrow (x+yi)^2 = -3-4i$

$\Leftrightarrow x^2 + 2xyi + y^2i^2 = -3-4i$

$\Leftrightarrow x^2 + 2xyi - y^2 = -3-4i$

$$\Leftrightarrow \begin{cases} x^2 - y^2 = -3 \\ 2xy = -4 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2 - y^2 = -3 \\ xy = -2 \end{cases}$$

$$\underset{x\neq0}{\Leftrightarrow} \begin{cases} y = \dfrac{-2}{x} \\ x^2 - \left(\dfrac{-2}{x}\right)^2 = -3 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = \dfrac{-2}{x} \\ x^4 - 4 = -3x^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = \dfrac{1}{x} \\ x^4 + 3x^2 - 4 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = \dfrac{-2}{x} \\ x^2 = 1 \end{cases} \quad \text{of} \quad \begin{cases} y = \dfrac{-2}{x} \\ \cancel{x^2 = -4} \end{cases}$$

$$\Leftrightarrow \begin{cases} y = -2 \\ x = 1 \end{cases} \quad \text{of} \quad \begin{cases} y = 2 \\ x = -1 \end{cases}$$

$\Leftrightarrow x = yi = 1-2i \quad$ of $\quad x + yi = -1+2i$

Antwoord: in $\mathbb{C}$ heeft 3−4i twee tegengestelde vierkantswortels: 1−2i en −1+2i d $5+12i$

$x+yi$ is een vierkantswortel van $5+12i$

$\Leftrightarrow (x+yi)^2 = 5+12i$

$\Leftrightarrow x^2+2xyi+y^2i^2 = 5+12i$

$\Leftrightarrow x^2+2xyi-y^2 = 5+12i$

$$\Leftrightarrow \begin{cases} x^2-y^2=5 \\ 2xy=12 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2-y^2=5 \\ xy=6 \end{cases}$$

$$\underset{x\neq 0}{\Leftrightarrow} \begin{cases} y=\dfrac{6}{x} \\ x^2-\left(\dfrac{6}{x}\right)^2=5 \end{cases}$$

$$\Leftrightarrow \begin{cases} y=\dfrac{6}{x} \\ x^4-36=5x^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} y=\dfrac{6}{x} \\ x^4-5x^2-36=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} y=\dfrac{6}{x} \\ x^2=9 \end{cases} \quad \text{of} \quad \begin{cases} y=\dfrac{6}{x} \\ \cancel{x^2=-4} \end{cases}$$

$$\Leftrightarrow \begin{cases} y=2 \\ x=3 \end{cases} \quad \text{of} \quad \begin{cases} y=-2 \\ x=-3 \end{cases}$$

$\Leftrightarrow x+yi = 3+2i$ of $x+yi=-3-2i$

Antwoord: in $\mathbb{C}$ heeft 5+12i twee tegengestelde vierkantswortels: 3+2i en −3−2i e $-i$

$x + yi$ is een vierkantswortel van $-i$

$\Leftrightarrow (x+yi)^2 = -i$

$\Leftrightarrow x^2 + 2xyi - y^2 = -i$

$\Leftrightarrow x^2 + 2xyi - y^2 = -i$

$$\Leftrightarrow \begin{cases} x^2 - y^2 = 0 \\ 2xy = -1 \end{cases}$$

$$\underset{x \neq 0}{\Leftrightarrow} \begin{cases} y = \frac{-1}{2x} \\ x^2 - \left(\frac{-1}{2x}\right)^2 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = \frac{-1}{2x} \\ 4x^4 - 1 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = \frac{-1}{2x} \\ x^2 = \frac{1}{2} \end{cases} \quad \text{of} \quad \begin{cases} y = \frac{-1}{2x} \\ \cancel{x^2 = -\frac{1}{2}} \end{cases}$$

$$\Leftrightarrow \begin{cases} y = -\frac{\sqrt{2}}{2} \\ x = \frac{\sqrt{2}}{2} \end{cases} \quad \text{of} \quad \begin{cases} y = \frac{\sqrt{2}}{2} \\ x = -\frac{\sqrt{2}}{2} \end{cases}$$

$$\Leftrightarrow x + y\,i = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i \quad \text{of} \quad x + y\,i = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$$

Antwoord: in $\mathbb{C}$ heeft $-i$ twee tegengestelde vierkantswortels: $\frac{\sqrt{2}}{2}(1-i)$ en $\frac{\sqrt{2}}{2}(-1+i)$

f $1+2i\sqrt{6}$

$x+yi$ is een vierkantswortel van $1+2\sqrt{6}\,i$

$\Leftrightarrow (x+yi)^2 = 1+2\sqrt{6}\,i$

$\Leftrightarrow x^2+2xyi-y^2 = 1+2\sqrt{6}\,i$

$\Leftrightarrow x^2+2xyi-y^2 = 1+2\sqrt{6}\,i$

$$\Leftrightarrow \begin{cases} x^2-y^2=1 \\ xy=\sqrt{6} \end{cases}$$

$$\underset{x\neq 0}{\Leftrightarrow} \begin{cases} y=\dfrac{\sqrt{6}}{x} \\ x^2-\left(\dfrac{\sqrt{6}}{x}\right)^2=1 \end{cases}$$

$$\Leftrightarrow \begin{cases} y=\dfrac{\sqrt{6}}{x} \\ x^4-6=x^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^4-x^2-6=0 \\ y=\dfrac{\sqrt{6}}{x} \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2=3 \\ y=\dfrac{\sqrt{6}}{x} \end{cases} \quad \text{of} \quad \begin{cases} \cancel{x^2=-2} \\ y=\dfrac{\sqrt{6}}{x} \end{cases}$$

$$\Leftrightarrow \begin{cases} y=\sqrt{2} \\ x=\sqrt{3} \end{cases} \quad \text{of} \quad \begin{cases} y=-\sqrt{2} \\ x=-\sqrt{3} \end{cases}$$

Antwoord:

in $\mathbb{C}$ heeft $1+2\sqrt{6}i$ twee tegengestelde vierkantswortels: $\sqrt{3}+\sqrt{2}\,i$ en $-\sqrt{3}-\sqrt{2}\,i$

g $48+14i$

$x+yi$ is een vierkantswortel van $48+14i$

$\Leftrightarrow (x+yi)^2 = 48+14i$

$\Leftrightarrow x^2+2xyi-y^2 = 48+14i$

$$\Leftrightarrow \begin{cases} x^2-y^2=48 \\ xy=14 \end{cases}$$

$$\underset{x\neq 0}{\Leftrightarrow} \begin{cases} y=\dfrac{7}{x} \\ x^2-\left(\dfrac{7}{x}\right)^2=48 \end{cases}$$

$$\Leftrightarrow \begin{cases} y=\dfrac{7}{x} \\ x^4-48=48x^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^4-48x^2-49=0 \\ y=\dfrac{7}{x} \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2=49 \\ y=\dfrac{7}{x} \end{cases} \quad \text{of} \quad \begin{cases} \cancel{x^2=-1} \\ y=\dfrac{7}{x} \end{cases}$$

$$\Leftrightarrow \begin{cases} x=7 \\ y=1 \end{cases} \quad \text{of} \quad \begin{cases} x=-7 \\ y=-1 \end{cases}$$

Antwoord:

in $\mathbb{C}$ heeft 48+14i twee tegengestelde vierkantswortels: $7+i$ en $-7-i$

h $-21+20i$

$x+yi$ is een vierkantswortel van $-21+20i$

$\Leftrightarrow (x+yi)^2 = -21+20i$

$\Leftrightarrow x^2+2xyi-y^2 = -21+20i$

$$\Leftrightarrow \begin{cases} x^2-y^2=-21 \\ xy=10 \end{cases}$$

$$\underset{x\neq 0}{\Leftrightarrow} \begin{cases} y=\dfrac{10}{x} \\ x^2-\left(\dfrac{10}{x}\right)^2=-21 \end{cases}$$

$$\Leftrightarrow \begin{cases} y=\dfrac{10}{x} \\ x^4-100=-21x^2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^4+21x^2-100=0 \\ y=\dfrac{10}{x} \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2=4 \\ y=\dfrac{10}{x} \end{cases} \quad \text{of} \quad \begin{cases} \cancel{x^2=-25} \\ y=\dfrac{10}{x} \end{cases}$$

$$\Leftrightarrow \begin{cases} x=2 \\ y=5 \end{cases} \quad \text{of} \quad \begin{cases} x=-2 \\ y=-5 \end{cases}$$

Antwoord:

in $\mathbb{C}$ heeft $-21+20i$ twee tegengestelde vierkantswortels: $2+5i$ en $-2-5i$

i $-i(96+28i)$

$x+yi$ is een vierkantswortel van $28-96i$

$\Leftrightarrow (x+yi)^2 = 28-96i$

$\Leftrightarrow x^2+2xyi-y^2 = 28-96i$

$$\Leftrightarrow \begin{cases} x^2-y^2 = 28 \\ xy = -48 \end{cases}$$

$$\underset{x\neq 0}{\Leftrightarrow} \begin{cases} y = \dfrac{-48}{x} \\ x^2 - \left(\dfrac{-48}{x}\right)^2 = 28 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = \dfrac{-48}{x} \\ x^4-28x^2-2304 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2 = 64 \\ y = \dfrac{-48}{x} \end{cases} \quad \text{of} \quad \begin{cases} \cancel{x^2 = -36} \\ y = \dfrac{-48}{x} \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 8 \\ y = -6 \end{cases} \quad \text{of} \quad \begin{cases} x = -8 \\ y = 6 \end{cases}$$

Antwoord:

in $\mathbb{C}$ heeft 28-96i twee tegengestelde vierkantswortels: 8-6i en $-8+6i$

j $(3-4i)(-8+6i)$

$x+yi$ is een vierkantswortel van $(3-4i)(-8+6i) = 50i$

$\Leftrightarrow (x+yi)^2 = 50i$

$\Leftrightarrow x^2+2xyi-y^2 = 50i$

$$\Leftrightarrow \begin{cases} x^2-y^2 = 0 \\ xy = 25 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = y \\ xy = 25 \end{cases} \quad \text{of} \quad \cancel{\begin{cases} x = -y \\ xy = 25 \end{cases}}$$

$$\Leftrightarrow \begin{cases} x = 5 \\ y = 5 \end{cases} \quad \text{of} \quad \begin{cases} x = 5 \\ y = -5 \end{cases}$$

Antwoord:

in $\mathbb{C}$ heeft 50i twee tegengestelde vierkantswortels: 5+5i en $-5-5i$

k $\quad \dfrac{-1}{5+12i}$

x + yi is een vierkantswortel van $\dfrac{-1}{5+12i}=\dfrac{-5+12i}{169}$

$169=13^2$, we bepalen de vierkantswortels uit de teller:

x+yi is een vierkantswortel van −5 + 12i

$$\Leftrightarrow (x+yi)^2=-5+12i$$

$$\Leftrightarrow x^2+2xyi-y^2=-5+12i$$

$$\Leftrightarrow \begin{cases} x^2-y^2=-5 \\ xy=6 \end{cases}$$

$$\underset{x\neq 0}{\Leftrightarrow} \begin{cases} y=\dfrac{6}{x} \\ x^2-\left(\dfrac{6}{x}\right)^2=-5 \end{cases}$$

$$\Leftrightarrow \begin{cases} y=\dfrac{6}{x} \\ x^4+5x^2-36=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2=4 \\ y=\dfrac{6}{x} \end{cases} \quad \text{of} \quad \begin{cases} \cancel{x^2=-9} \\ y=\dfrac{6}{x} \end{cases}$$

$$\Leftrightarrow \begin{cases} x=2 \\ y=3 \end{cases} \quad \text{of} \quad \begin{cases} x=-2 \\ y=-3 \end{cases}$$

Antwoord :

in $\mathbb{C}$ heeft $\dfrac{-1}{5+12i}$ twee tegengestelde vierkantswortels: $\dfrac{2+3i}{13}$ en $\dfrac{-2-3i}{13}$

l $\frac{54+62i}{1+3i}$

$x+yi$ is een vierkantswortel van $\frac{54+62i}{1+3i}=24-10i$

$$(x+yi)^2=24-10i$$

$$\Leftrightarrow x^2+2xyi-y^2=24-10i$$

$$\Leftrightarrow \begin{cases} x^2-y^2=24 \\ xy=-5 \end{cases}$$

$$\underset{x\neq 0}{\Leftrightarrow} \begin{cases} y=\frac{-5}{x} \\ x^2-\left(\frac{-5}{x}\right)^2=24 \end{cases}$$

$$\Leftrightarrow \begin{cases} y=\frac{-5}{x} \\ x^4-24x^2-25=0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x^2=25 \\ y=\frac{-5}{x} \end{cases} \quad \text{of} \quad \begin{cases} \cancel{x^2=-1} \\ y=\frac{-5}{x} \end{cases}$$

$$\Leftrightarrow \begin{cases} x=5 \\ y=-1 \end{cases} \quad \text{of} \quad \begin{cases} x=-5 \\ y=1 \end{cases}$$

Antwoord :

in $\mathbb{C}$ heeft $\frac{54+62i}{1+3i}$ twee tegengestelde vierkantswortels: 5-i en -5+i

Oef 10 blz 30

Los op in $\mathbb{C}$.

a $z^2-6z+10=0$

$$D=(-6)^2-4\cdot 1\cdot 10=-4$$

$$\Leftrightarrow z=\frac{-(-6)+2i}{2}=3+i \quad \text{of} \quad z=\frac{6-2i}{2}=3-i$$

$$V=\{3+i,3-i\}$$

b $z^2-4z+13=0$

$$D=(-4)^2-4\cdot 1\cdot 13=-36$$

$$\Leftrightarrow z=\frac{4+6i}{2}=2+3i \quad \text{of} \quad z=\frac{4-6i}{2}=2-3i$$

$$V=\{2+3i,2-3i\}$$

c $z^2+(i-5)z=7i-26$

$\Leftrightarrow z^2+(i-5)z-7i+26=0$

$D=(i-5)^2-4\cdot1\cdot(-7i+26)=-80+18i$

De vierkantswortels uit −80+18i zijn $\pm(1+9i)$ zodat: $z=\dfrac{-i+5\pm(1+9i)}{5}=\left\langle\begin{array}{l}3+4i\\2-5i\end{array}\right.$

$V=\{3+4i;2-5i\}$

d $iz^2+2z-13i-16=0$

$D=2^2-4\cdot i\cdot(-13i-16)=-48+64i$

De vierkantswortels uit −48 + 64 i zijn $\pm(4+8i)$ zodat: $z=\dfrac{-2\pm(4+8i)}{2}=\left\langle\begin{array}{l}4-i\\-4+3i\end{array}\right.$

$V=\{4-i;-4+3i\}$

e $z^2-4z+7+4i=0$

$D=(-4)^2-4\cdot1\cdot(7+4i)=-12-16i$

De vierkantswortels uit −12−16i zijn $\pm(2-4i)$ zodat: $z=\dfrac{4\pm(2-4i)}{2}=\left\langle\begin{array}{l}3-2i\\1+2i\end{array}\right.$

$V=\{3-2i;1+2i\}$

f $z^2+(i-z)z+5(1-i)=0$

$D=(i-4)^2-4\cdot1\cdot5(1-i)=-5+12i$

De vierkantswortels uit −5+12i zijn $\pm(2+3i)$ zodat: $z=\dfrac{-i+4\pm(2+3i)}{2}=\left\langle\begin{array}{l}3+i\\1-2i\end{array}\right.$

$V=\{3+i;1-2i\}$

g $z^2-(6+i)z+7+9i=0$

$D=(-6-i)^2-4\cdot1\cdot(7+9i)=7-24i$

De vierkantswortels uit 7−24i zijn $\pm(4-3i)$ zodat: $z=\dfrac{6+i\pm(4-3i)}{2}=\left\langle\begin{array}{l}5-i\\1+2i\end{array}\right.$

$V=\{5-i;1+2i\}$

h $z^2-(1+i)z+2(1+i)=0$

$$D=(-1-i)^2-4\cdot1\cdot(2+2i)=-8-6i$$

De vierkantswortels uit $-8-6i$ zijn$\pm(1-3i)$ zodat: $z=\dfrac{1+i\pm(1-3i)}{2}=\left\langle\begin{matrix}1-i\\2i\end{matrix}\right.$

$V=\{1-i;2i\}$

i $z^2-(4+2i)z+3+4i=0$

$$D=(-4-2i)^2-4\cdot1\cdot(3+4i)=0$$

Dus : $z=\dfrac{-(-4-2i)}{2\cdot1}=2+i$

$V=\{2+i\}$

j $z^3-2z^2+(16+8i)z=0$

$\Leftrightarrow z\left(z^2-2z+16+8i\right)=0$

$\Leftrightarrow z=0$ of $z^2-2z+16+8i=0$

$$D=(-2)^2-4\cdot1\cdot(16+8i)=-60-32i$$

De vierkantswortels uit $-60-32i$ zijn$\pm(2-8i)$ zodat: $z=\dfrac{2\pm(2-8i)}{2}=\left\langle\begin{matrix}2-4i\\4i\end{matrix}\right.$

$V=\{0;2-4i;4i\}$

k $z^4+(1-4i)z^2=12+16i$

stel $z^2=t$

$\Leftrightarrow t^2+(1-4i)t-12-16i=0$

$$D=(1-4i)^2-4\cdot1\cdot(-12-16i)=33+56i$$

De vierkantswortels uit $33+56$ i zijn$\pm(7+4i)$ zodat: $t=\dfrac{-1+4i\pm(7+4i)}{2}=\left\langle\begin{matrix}3+4i\\-4\end{matrix}\right.$

$\Leftrightarrow z^2=3+4i$ of $z^2=-4$

$\Leftrightarrow z=\pm(2+i)$ of $z=\pm2i$

$V=\{2+i;-2-i;2i;-2i\}$

l $z^4 + z^2 + 1 = 0$

stel $z^2 = t$

$\Leftrightarrow t^2 + t + 1 = 0$

$D = 1 - 4 = -3$

$\Leftrightarrow t = \dfrac{-1 \pm \sqrt{3}\,i}{2}$

$\Leftrightarrow z^2 = \dfrac{-1 + \sqrt{3}\,i}{2}$ of $z^2 = \dfrac{-1 - \sqrt{3}\,i}{2}$

$\Leftrightarrow z = \pm\dfrac{1 + \sqrt{3}\,i}{2}$ of $z = \pm\dfrac{1 - \sqrt{3}\,i}{2}$

$V = \left\{\pm\dfrac{1 + i\sqrt{3}}{2}; \pm\dfrac{1 - i\sqrt{3}}{2}\right\}$

m $z^4 + (1 - i)z^2 - i = 0$

stel $z^2 = t$

$\Leftrightarrow t^2 + (1 - i)t - i = 0$

$D = (1 - i)^2 = 4 \cdot 1 \cdot (-i) = 2i$

De vierkantswortels uit 2 i zijn $\pm(1+i)$ zodat: $t = \dfrac{-1 + i \pm (1 + i)}{2} = \left\langle \begin{matrix} i \\ -1 \end{matrix} \right.$

$\Leftrightarrow z^2 = i$ of $z^2 = -1$

$\Leftrightarrow z = \pm\dfrac{\sqrt{2}}{2}(1 + i)$ of $z = \pm i$

$V = \left\{\pm\dfrac{\sqrt{2}}{2}(1 + i);\ \pm i\right\}$

n $z^4 = \left(1 + 2i\sqrt{3}\right)z^2 + 2 + 2i\sqrt{3}$

stel $z^2 = t$

$\Leftrightarrow t^2 - \left(1 + 2\sqrt{3}\,i\right)t - 2 - 2i\sqrt{3} = 0$

$D = \left(-1 - 2\sqrt{3}\,i\right)^2 - 4 \cdot 1 \cdot \left(-2 - 2\sqrt{3}\,i\right) = -3 + 12\sqrt{3}\,i$

De vierkantswortels uit $-3 + 12\sqrt{3}\,i$ zijn $\pm\left(3 + 2\sqrt{3}\,i\right)$ zodat: $t = \dfrac{\left(1 + 2\sqrt{3}\,i\right) \pm \left(3 + 2\sqrt{3}\,i\right)}{2} = \left\langle \begin{matrix} 2 + 2\sqrt{3}\,i \\ -1 \end{matrix} \right.$

$\Leftrightarrow z^2 = 2 + 2\sqrt{3}\,i$ of $z^2 = -1$

$\Leftrightarrow z = \pm\left(\sqrt{3} + i\right)$ of $z = \pm i$

$V = \left\{\pm\left(\sqrt{3} + i\right); \pm i\right\}$

o $iz^2-(5+4i)z-8+14i=0$

$$D=(-5-4i)^2-4i(-8+14i)=65+72i=(9+4i)^2$$

$$\Leftrightarrow z=\frac{5+4i\pm(9+4)i}{2i}=\left\langle\begin{array}{l}4-7i\\ 2i\end{array}\right.$$

$$V=\{4-7i;2i\}$$

p $(2+i)z^2-(7+11i)z+1+38i=0$

$$D=(-7-11i)^2-4(2+i)(1+38i)=72-154i=(11-7i)^2$$

$$\Leftrightarrow z=\frac{7+11i\pm(11-7i)i}{2i}=\left\langle\begin{array}{l}4-i\\ 1+4i\end{array}\right.$$

$$V=\{4-i;1+4i\}$$

Oef 11 blz 30

Gegeven: $f(z)=\dfrac{z^2-z}{z+1}$

Bereken:

a $f(i)$

$$=\frac{i^2-i}{i+1}=\frac{-1-i}{i+1}=-1$$

f $f(2i-1)$

$$=\frac{(2i-1)^2-(2i-1)}{2i-1+1}$$

$$=\frac{4i^2-4i+1-2i+1}{2i}$$

$$=\frac{-2-6i}{2i}$$

$$=i-3$$

c $f\left(\dfrac{1+i}{1-i}\right)$

$$=f\left(\frac{(1+i)^2}{(1-i)(1+i)}\right)=f\left(\frac{1+2i+i^2}{1-i2}\right)=f\left(\frac{2i}{2}\right)=f(i)=\frac{i^2-i}{i+1}=\frac{-1-i}{i+1}=-1$$

d $f^{-1}(-3-i)$

$$\frac{z^2-z}{z+1}=-3-i$$

$$\Leftrightarrow z^2-z=(-3-i)(z+1)$$

$$\Leftrightarrow z^2+(2+i)z+3+i=0$$

$$D=(2+i)^2-4\cdot1\cdot(3+i)=-9$$

De vierkantswortels uit -9 zijn $\pm3i$ zodat: $z=\dfrac{-2-i\pm3i}{2}=\left\langle\begin{array}{l}-1-2i\\ -1+i\end{array}\right.$

e $f^{-1}(-5)$

$$\frac{z^2-z}{z+1}=-5$$

$$\Leftrightarrow z^2-z=-5(z+1)$$

$$\Leftrightarrow z^2+4z+5=0$$

$$D=-4$$

De vierkantswortels uit -4 zijn $\pm2i$ zodat: $z=\dfrac{-4\pm2i}{2}=\left\langle\begin{array}{l}-2-i\\ -2+i\end{array}\right.$

f $f^{-1}\left(\dfrac{-8-6i}{5}\right)$

$$\frac{z^2-z}{z+1}=\frac{-8-6i}{5}$$

$$\Leftrightarrow 5(z^2-z)=(z+1)(-8-6i)$$

$$\Leftrightarrow 5z^2+(3+6i)z+8+6i=0$$

$$\begin{aligned}D&=(3+6i)^2-4\cdot5\cdot(8+6i)\\&=-187-84i\\&=(3-14i)^2\end{aligned}$$

$$\Leftrightarrow z=\frac{-3-6i\pm(3-14i)}{10}=\left\langle\begin{array}{l}-0,6+0,8i\\ -2i\end{array}\right.$$

Oef 12 blz 31

Gegeven : $f(z) = z^2 + iz$

$g(z) = 2z + \overline{z}$

Bereken :

a $f(2-i) - 2g(1+3i)$

$= (2-i)^2 + i(2-i) - 2\left[2(1+3i) + \overline{1+3i}\right]$

$= 4 - 2i - 2(3+3i)$

$= -2 - 8i$

b $f(-2i) \cdot g(3)$

$= \left[(-2i)^2 + i(-2i)\right] \cdot \left[2 \cdot 3 + \overline{3}\right]$

$= (-4+2)(6+3)$

$= -18$

c $g\left(f(-1)\right)$

$f(-i) = (-i)^2 + i(-i) = -1 + 1 = 0$

$g\left(f(i)\right) = g(0) = 2 \cdot 0 + \overline{0} = 0$

d $f\left(g(1+2i)\right)$

$g(1+2i) = 2(1+2i) + \overline{(1+2i)}$

$= 2 + 4i + 1 - 2i$

$= 3 + 2i$

$f\left(g(1+2i)\right) = f(3+2i)$

$= (3+2i)^2 + i(3+2i)$

$= 5 + 12i + 3i - 2$

$= 3 + 15i$

e $f\left(f(1)\right)$

$f(1) = 1 + i$

$f\left(f(1)\right) = f(1+i)$

$= (1+i)^2 + i(1+i)$

$= 2i + i - 1$

$= -1 + 3i$

f $g\left(g\left(g(g(i))\right)\right)$

$$g(i) = 2i - \bar{i} = i$$

$$g\left(g(i)\right) = g(i) = i$$

$$g\left(g\left(g(i)\right)\right) = g(i) = i$$

$$g\left(g\left(g\left(g(i)\right)\right)\right) = g(i) = i$$

Oef 13 blz 31

Gegeven : $f(z) = \frac{1}{z}$

$g(z) = \bar{z}$

$h(z) = iz$

Bepaal het voorschrift van de complexe functies k, l, m en n met:

a $k(z) = \left(f \circ g \circ h\right)(z)$

$$h(z) = iz$$

$$g\left(h(z)\right) = \overline{iz} = -i\bar{z}$$

$$k(z) = f\left(g\left(h(z)\right)\right) = \frac{i}{-i\bar{z}} = -\frac{1}{\bar{z}}$$

b $l(z) = \left(f \circ h \circ g\right)(iz)$

$$g(iz) = \overline{iz} = -i\bar{z}$$

$$h\left(g(iz)\right) = h(-i\bar{z}) = \bar{z}$$

$$l(z) = f\left(h\left(g(iz)\right)\right) = \frac{i}{\bar{z}}$$

c $m(z) = \left(h \circ f \circ g\right)\left(\bar{z}\right)$

$$g(\bar{z}) = \bar{\bar{z}} = z$$

$$f\left(g(\bar{z})\right) = \frac{i}{z}$$

$$m(z) = h\left(f\left(g(iz)\right)\right) = i \cdot \frac{i}{z} = \frac{-1}{z}$$

d $n(z) = \left(g \circ h \circ f\right)\left(-\frac{1}{z}\right)$

$$f\left(\frac{-1}{z}\right) = \frac{i}{-\frac{1}{z}} = -iz$$

$$h\left(f\left(-\frac{1}{2}\right)\right) = i(iz) = z$$

$$n(z) = g\left(h\left(f\left(-\frac{1}{2}\right)\right)\right) = \bar{z}$$

Oef 14 blz 31

Bepaal $\lambda \in \mathbb{C}$ zodat de volgende vierkantsvergelijkingen twee identieke wortels heeft; Bepaal deze wortels.

$iz^2 + (\lambda - 3i)z - 2 = 0$

$z_1 = z_2$

$\Leftrightarrow D = 0$

$\Leftrightarrow (\lambda - 3i)^2 - 4i(-2) = 0$

$\Leftrightarrow \lambda^2 - 6i\lambda + 9i^2 + 8i = 0$

$\Leftrightarrow \lambda^2 - 6i\lambda + 9i^2 + 8i = 0$

$\Leftrightarrow \lambda^2 - 6i\lambda - 9 + 8i = 0$

$\underset{ICT}{\Leftrightarrow} \lambda = -2 + 5i \qquad \text{of} \qquad \lambda = 2 + i$

- $\lambda = -2 + 5i$

  VKV : $iz^2 + (-2 + 5i - 3i)z - 2 = 0$

  $\Leftrightarrow iz^2 + (-2 + 2i)z - 2 = 0$

  $\underset{ICT}{\Leftrightarrow} z = -1 - i$

  $V = \{-1 - i\}$

- $\lambda = 2 + i$

  VKV : $iz^2 + (2 + i - 3i)z - 2 = 0$

  $\Leftrightarrow iz^2 + (2 - 2i)z - 2 = 0$

  $\underset{ICT}{\Leftrightarrow} z = 1 + i$

  $V = \{1 + i\}$

Oef 15 blz 31

Bepaal $\lambda \in \mathbb{C}$ zodat de volgende vierkantsvergelijkingen twee tegengestelde wortels heeft; Bepaal deze wortels.

$(1+i)z^2 + (\lambda - 4 + i)z + 22 - 2\lambda = 0$

de vergelijking heeft twee tegengestelde wortels $\Leftrightarrow$ som van de wortels is 0

$\Leftrightarrow -\frac{\lambda - 4 + i}{1 + i} = 0$

$\Leftrightarrow \lambda - 4 + i = 0$

$\Leftrightarrow \lambda = 4 - i$

De vierkantsvergelijking wordt:

$(1+i)z^2 + 22 - 2(4 - i)$

$\Leftrightarrow (1+i)z^2 = -14 - 2i$

$\Leftrightarrow z = 1 + 3i \quad \text{of} \quad z = -1 - 3i$

Oef 16 blz 31

Bepaal $\lambda \in \mathbb{C}$ zodat de volgende vierkantsvergelijkingen twee tegengestelde wortels heeft; Bepaal deze wortels.

$z^2 + 2(5i - \lambda)z + 2i(3 - \lambda) = 0$

stel $\lambda = a + bi$

de vergelijking heeft twee complex toegevoegde wortels $\Leftrightarrow$ de som en product reëel zijn

$$\begin{cases} 2(5i - a - bi) \text{ is reëel} \\ 2i(3 - a - bi) \text{ is reëel} \end{cases}$$

$$\Leftrightarrow \begin{cases} 5 - b = 0 \\ 3 - a = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} a = 3 \\ b = 5 \end{cases}$$

$$\Rightarrow \lambda = 3 + 5i$$

de vergelijking wordt:

$$z^2 + 6\lambda + 10 = 0$$

$$\Leftrightarrow z = -3 - i \quad \text{of} \quad z = -3 + i$$

Oef 17 blz 31

Ga na voor welke x, y $\in$ IR onderstaande gelijkheden gelden.

a $\quad (x + y) + (3x - y)i = (2x - y) + (x + y)i$

$$\Leftrightarrow x = y + 3xi - yi = 2x - y + xi + yi$$

$$\Leftrightarrow -x + 2y + (2x - 2y)i = 0$$

$$\Leftrightarrow \begin{cases} -x + 2y = 0 \\ x - y = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$$

b $\quad (-1 + 2i)(x + yi) - 3(x + yi) = (5 + 2i)i$

$$\Leftrightarrow -x - yi + 2xi - 2y - 3x - 3yi = 5i - 2$$

$$\Leftrightarrow -4x - 2y + 2 + (2x - 4y - 5)i = 0$$

$$\Leftrightarrow \begin{cases} 2x + y = 1 \\ 2x - 4y = 5 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = \dfrac{9}{10} \\ y = -\dfrac{4}{5} \end{cases}$$

c $\quad (x+y\,i)^2-(x-y\,i)^2=0$

$\Leftrightarrow x+y\,i=x-y\,i \quad$ of $\quad x+y\,i=-x+y\,i$

$\Leftrightarrow y=0 \quad$ of $\quad x=0$

$V=\{(0,k),(k,0)\mid k\in IR\}$

Oef 18 blz 32

De vergelijking $z^2+az+b=0\ (a,b\in IR)$ heeft als wortel 2 – i.

Bepaal a en b en de andere wortel.

$2-i$ is een oplossing van $z^2+az+b=0$

$\Leftrightarrow (2-i)^2+a(2-i)+b=0$

$\Leftrightarrow 4-4i+i^2+2a-ai+b=0$

$\Leftrightarrow (3+2a+b)-(4+a)i=0$

$$\Leftrightarrow \begin{cases}3+2a+b=0\\ 4+a=0\end{cases}$$

$$\Leftrightarrow \begin{cases}a=-4\\ b=5\end{cases}$$

$\text{VKV}: z^2-4z+5=0$

$\underset{ICT}{\Leftrightarrow} z=2-i \quad$ of $\quad z=2+i$

Oef 19 blz 32

Voor welke reële $k\neq 0$ heeft de vergelijking $(2-3i)z^2-(k-1)z+4+3i=0$ een reële wortel?

Bereken voor deze k alle wortels van de vergelijking.

a is een reële wortel van de vergelijking

$\Leftrightarrow (2-3i)a^2-(k-1)a+4+3i=0$

$\Leftrightarrow \left(2a^2-ka+a+4\right)+\left(-3a^2+3\right)i=0$

$$\Leftrightarrow \begin{cases}2a^2-ka+a+4=0\\ -3a^2+3=0\end{cases}$$

$$\Leftrightarrow \begin{cases}a=1\\ k=7\end{cases} \quad \text{of} \quad \begin{cases}a=-1\\ k=-5\end{cases}$$

$\text{VKV}: \bullet\ (2-3i)z^2-6z+4+3i=0$

$\underset{ICT}{\Leftrightarrow} z=1 \quad$ of $\quad z=-\frac{1}{13}+\frac{18}{13}i$

$\bullet\ (2-3i)z^2+6z+4+3i=0$

$\underset{ICT}{\Leftrightarrow} z=-1 \quad$ of $\quad z=\frac{1}{13}-\frac{18}{13}i$

Oef 20 blz 32

Bepaal alle $z \in \mathbb{C}$ waarvoor: $z^2 + 2\overline{z} + 6 = 0$.

stel $z = x + yi$ $\quad(x, y \in \mathrm{IR})$

$$\Leftrightarrow (x + yi)^2 + 2\overline{(x + yi)} + 6 = 0$$

$$\Leftrightarrow x^2 + 2xyi - y^2 + 2x - 2yi + 6 = 0$$

$$\Leftrightarrow \begin{cases} x^2 - y^2 + 2x + 6 = 0 \\ xy - y = 0 \end{cases}$$

$$\Leftrightarrow \cancel{\begin{cases} y = 0 \\ x^2 + 2x + 6 = 0 \end{cases}} \quad \text{of} \quad \begin{cases} x = 1 \\ 1 - y^2 + 2 + 6 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 1 \\ y = 3 \end{cases} \quad \text{of} \quad \begin{cases} x = 1 \\ y = -3 \end{cases}$$

Hieruit volgt: $z = 1 + 3i$ of $z = 1 - 3i$

$V = \{1 + 3i,\ 1 - 3i\}$

Oef 21 blz 32

Bepaal alle $z \in \mathbb{C}$ waarvoor $2z^2 - \overline{z}^2 + 12z + 20 = 0$

stel $z = x + yi$ $\quad(x, y \in \mathrm{IR})$

$$\Leftrightarrow 2(x + y\,i)^2 - \overline{(x + y\,i)} + 12(x + y\,i) + 20 = 0$$

$$\Leftrightarrow 2\left(x^2 + 2xy\,i - y^2\right) - \left(x^2 - 2xy\,i - y^2\right) + 12x + 12y\,i + 20 = 0$$

$$\Leftrightarrow \left(x^2 - y^2 + 12x + 20\right) + (6xy + 12y)i = 0$$

$$\Leftrightarrow \begin{cases} x^2 - y^2 + 12x + 20 = 0 \\ xy + 2y = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = 0 \\ x^2 + 12x + 20 = 0 \end{cases} \quad \text{of} \quad \begin{cases} x = -2 \\ 4 - y^2 - 24 + 20 = 0 \end{cases}$$

$$\Leftrightarrow \begin{cases} y = 0 \\ x = -2 \end{cases} \quad \text{of} \quad \begin{cases} y = 0 \\ x = -10 \end{cases} \quad \text{of} \quad \begin{cases} x = -2 \\ y = 0 \end{cases}$$

Hieruit volgt: $z = -2 + 0 \cdot i$ of $z = -10 + 0 \cdot i$

$V = \{-2,\ -10\}$

Oef 22 blz 32

Bewijs volgende eigenschappen.

a $\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$

$$\begin{aligned}
\overline{\left(\frac{z_1}{z_2}\right)} &= \overline{\left(\frac{x_1 + y_1 i}{x_2 + y_2 i}\right)} \\
&= \overline{\left(\frac{(x_1 + y_1 i)(x_2 - y_2 i)}{(x_2 + y_2 i)(x_2 - y_2 i)}\right)} \\
&= \overline{\left(\frac{(x_1 x_2 + y_1 y_2) + (-x_1 y_2 + y_1 x_2) i}{x_2^2 + y_2^2}\right)} \\
&= \frac{(x_1 x_2 + y_1 y_2) + (x_1 y_2 - y_1 x_2) i}{x_2^2 + y_2^2} \qquad (1)
\end{aligned}$$

$$\begin{aligned}
\frac{\overline{z_1}}{\overline{z_2}} &= \frac{\overline{x_1 + y_1 i}}{\overline{x_2 + y_2 i}} \\
&= \frac{x_1 - y_1 i}{x_2 - y_2 i} \\
&= \frac{(x_1 - y_1 i)(x_2 + y_2 i)}{(x_2 - y_2 i)(x_2 + y_2 i)} \\
&= \frac{(x_1 x_2 + y_1 y_2) + (x_1 y_2 - y_1 x_2) i}{x_2^2 + y_2^2} \qquad (2)
\end{aligned}$$

Uit (1) en (2) volgt het gevraagde.

b $\mathrm{Re}(z) = \frac{z + \overline{z}}{2}$

$$\frac{z + \overline{z}}{2} = \frac{x + y i + \overline{x + y i}}{2} = \frac{x + y i + x - y i}{2} = x = \mathrm{Re}(z)$$

c $\mathrm{Im}(z) = \frac{z - \overline{z}}{2i}$

$$\frac{z - \overline{z}}{2i} = \frac{x + y i - (\overline{x + y i})}{2i} = \frac{x + y i - x + y i}{2i} = y = \mathrm{Im}(z)$$

Oef 23 blz 32

Toon aan dat als z = x + yi een oplossing is van de vierkantsvergelijking $az^2 + bz + c = 0$ met $a \in \mathrm{IR}_0$ en $b, c \in \mathrm{IR}$,
Dan ook $\overline{z} = x - y\,i$ een oplossing is van de vierkantsvergelijking.

$z = x + y\,i$ is een oplossing van $az^2 + bz + c = 0$

$$\Leftrightarrow a(x + y\,i)^2 + b(x + y\,i) + c = 0$$

$$\Leftrightarrow a(x^2 - y^2 + 2xy\,i) + b(x + y\,i) + c = 0$$

$$\Leftrightarrow \left[a\left(x^2 - y^2\right) + bx + c\right] + \left[2axy + by\right]i = 0$$

$$\Leftrightarrow \begin{cases} a\left(x^2 - y^2\right) + bx + c = 0 \\ 2axy + by = 0 \end{cases}$$

$$\Leftrightarrow \left[a\left(x^2 - y^2\right) + bx + c\right] - \left[2axy + by\right]i = 0$$

$$\Leftrightarrow a\left(x^2 - y^2 - 2xyi\right) + b(x - yi) + c = 0$$

$$\Leftrightarrow a(x - yi)^2 + b(x - yi) + c = 0$$

$\Leftrightarrow \overline{z} = x - y\,i$ is een oplossing van $az^2 + bz + c = 0$

Oef 24 blz 32

Bereken $\dfrac{(1-3\,i)^{3n}}{(1+i)^n}$ met $n \in \mathrm{IN}_0$.

$$\begin{aligned}\frac{(1-i)^{3n}}{(1+i)^n} &= \left(\frac{1+3\,(-i)+3\,(-i)^2+(-i)^3}{1+i}\right)^n \\ &= \left(\frac{1-3\,i-3+i}{1+i}\right)^n \\ &= \left(\frac{-2\,(1+i)}{1+i}\right)^n \\ &= (-2)^n\end{aligned}$$

Oef 25 blz 32

Toon aan: $x = \dfrac{1-i\sqrt{3}}{2} \Rightarrow \dfrac{1}{x^2 - x} = -1$

$$\begin{aligned}\frac{1}{x^2 - x} = \frac{1}{x\left(x-1\right)} &= \frac{1}{\left(\frac{1-i\sqrt{3}}{2}\right)\left(\frac{1-i\sqrt{3}}{2} - 1\right)} \\ &= \frac{1}{\left(\frac{1-i\sqrt{3}}{2}\right)\left(\frac{-1-i\sqrt{3}}{2}\right)} \\ &= \frac{4}{-1+3i^2} = \frac{4}{-4} = -1\end{aligned}$$